ARL 158

THE FLOW OF AN INVISCID COMPRESSIBLE CONDUCTING FLUID PAST A SLENDER BODY OF AN ARBITRARY CROSS SECTION

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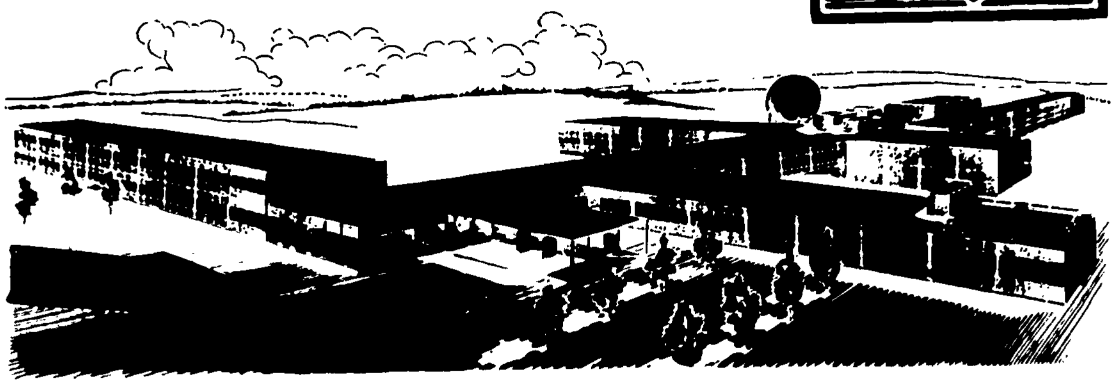

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Unclassified Report

The problems in magnetohydrodynamics, in which an inviscid compressible fluid with small conductivity flows steadily past a slender body of an arbitrary cross section, in the presence of an applied magnetic field parallel to the uniform flow are considered. By the use of the slender body approximation we shall discuss the character of the

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FOREWORD

This technical report was prepared by Rensselaer Polytechnic Institute, Troy, New York, on contract AF 33(616)-7312 for the Aeronautical Research Laboratory, Office of Aerospace Research. The work was accomplished on task 70169, "Thermo-Aerodynamic Characteristics at Hypersonic Mach Numbers" of Project 7064, "Research on Aerodynamic Flow Fields". The research work reported herein was carried out during the time Dr. Ken-ichi Kusukawa was on leave of absence from Tokyo Metropolitan University.

Mr. Anthony Fiore of the Hypersonic Research Branch, ARL, was the contract monitor.

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ABSTRACT

The problems in magnetohydrodynamics, in which an inviscid compressible fluid with small conductivity flows steadily past a slender body of an arbitrary cross section, in the presence of an applied magnetic field parallel to the uniform flow are considered. By the use of the slender body approximation we shall discuss the character of the velocity and the magnetic fields, and obtain the drag and the lateral force exerted on the body. For examples, the flows past a body of an elliptic cross section and a body of revolution at a small incidence will be discussed. It is noticeable that the induced drag for a slender body of revolution with pointed nose and tail ends is negative, and the lift for the same body takes the negative sign for a positive incidence.

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INTRODUCTION

Recently Sears and Resler (1) investigated the flow of a non-viscous incompressible fluid with an electrical conductivity past a two-dimensional thin obstacle by the linearized theory. In their paper, the conductivity of fluid is supposed to be infinite or very high. The same problem for an arbitrary conductivity was studied by McCune (2). The correlation effect between the compressibility and the conductivity was studied by Taniuchi (3), Kogan (4), McCune and Resler (5), and Sears (6). In these papers, the conductivity of fluid is supposed to be infinite. The flows of the non-viscous compressible fluid with a small conductivity past a two-dimensional thin body, and a body of revolution at zero incidence were studied by Sakurai (7), (8), and Kusukawa (9), respectively. Sakurai and Kusukawa take the magnetohydrodynamic Stokes' and Oseen's approximations, respectively. Resler and McCune (10) studied the flow of a compressible fluid with an arbitrary conductivity past a sinusoidal wall. Ando (11), (12), discussed the character of the linearized equation for the flow of a compressible fluid with an arbitrary conductivity past a thin body, in the presence of the uniform applied magnetic field in an arbitrary direction.

In the present paper we shall study the flow of a non-viscous compressible fluid with a low conductivity past a slender body of an arbitrary cross section, in the presence of a uniform applied magnetic field parallel to the uniform flow. We shall linearize the fundamental equation, and apply the slender body theory (13) to the present magnetohydrodynamic flow. The induced electric field is quasi-two-dimensional in the sectional plane. Both the velocity and the magnetic fields can be decomposed into the axial field and the irrotational two-dimensional field in the sectional plane. We shall discuss the circulation of fluid. The total force exerted on the body will be calculated. For an example, the drag and the lift for a body of revolution with pointed nose and tail ends at a small incidence will be studied. It is worthwhile to notice here that the induced drag is negative, and the lift is negative for a positive incidence.

I FUNDAMENTAL EQUATIONS FOR FLUID FLOW

We shall consider the flow of a non-viscous compressible fluid with a low electrical conductivity σ^* past a body of an arbitrary cross section, in the presence of an applied uniform magnetic field H_0^* parallel to the uniform flow U_0^*. We shall take the cms electromagnetic system of units and the cartesian coordinates x*, y*, s*, where s* is parallel to the uniform flow. For simplicity, we shall normalize the physical quantities with respect to the length L_0^* of the body, the uniform velocity U_0^*, the uniform magnetic field H_0^*, and the density ρ_0^* and the temperature T_0^* in the uniform flow such that

$$x = \frac{x^*}{L_0^*} \quad y = \frac{y^*}{L_0^*} \quad s = \frac{s^*}{L_0^*} \quad u = \frac{u^*}{U_0^*} \quad v = \frac{v^*}{U_0^*} \quad w = \frac{w^*}{U_0^*}$$

$$h_x = \frac{h_x^*}{H_0^*} \quad h_y = \frac{h_y^*}{H_0^*} \quad h_s = \frac{h_s^*}{H_0^*} \quad e_x = \frac{e_x^*}{H_0^* U_0^*} \quad e_y = \frac{e_y^*}{H_0^* U_0^*} \quad e_s = \frac{e_s^*}{H_0^* U_0^*}$$

$$j_x = \frac{4\pi j_x^* L_0^*}{H_0^*} \qquad j_y = \frac{4\pi j_y^* L_0^*}{H_0^*} \qquad j_s = \frac{4\pi j_s^* L_0^*}{H_0^*}$$

$$p = \frac{p^*}{\rho_0^* U_0^{*2}} \qquad \rho = \frac{\rho^*}{\rho_0^*} \qquad T = \frac{T^*}{T_0^*}$$

where u, v, w, denote the components of the perturbation velocity, h_x, h_y, h_s the perturbation magnetic field, e_x, e_y, e_s the induced electric field, j_x, j_y, j_s, the induced electric current density, p the pressure, ρ the density, T the temperature, and the notations with and without asterisk represent the physical and the dimensionless quantities, respectively. For convenience we shall introduce the dimensionless parameters defined by

$$S = \frac{H_0^{*2}}{4\pi\rho_0^* U_0^{*2}} \qquad R_m = 4\pi\sigma^* U_0^* L_0^*$$

$$Q = SR_m = \frac{\sigma^* H_0^{*2} U_0^* L_0^*}{\rho_0^* U_0^{*2}}$$

where S and Rm are the pressure number and the magnetic Reynolds number, respectively.

Supposing that the magnetic permeabilities of both the body and the fluid are unity, and the fluid is non-viscous and thermally non-conducting, we can write the equation of motion, the equation of continuity, the Ohm's law, the Maxwell's equation, and the conservation of energy in the following form:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+(1+w)\frac{\partial u}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{S}{\rho}\left\{j_y(1+h_s)-j_s h_y\right\} \tag{1a}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+(1+w)\frac{\partial v}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{S}{\rho}\left\{j_s h_x-j_x(1+h_s)\right\} \tag{1b}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+(1+w)\frac{\partial w}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial s}=\frac{S}{\rho}\left\{j_x h_y-j_y h_x\right\} \tag{1c}$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial s}(\rho+\rho w)=0 \tag{2}$$

$$j_x=R_m\left\{v(1+h_s)-(1+w)h_y+e_x\right\} \tag{3a}$$

$$j_y=R_m\left\{(1+w)h_x-u(1+h_s)+e_y\right\} \tag{3b}$$

$$j_s=R_m\left\{u h_y-v h_x+e_s\right\} \tag{3c}$$

$$\frac{\partial h_s}{\partial y}-\frac{\partial h_y}{\partial s}=j_x \tag{4a}$$

$$\frac{\partial h_x}{\partial s}-\frac{\partial h_s}{\partial x}=j_y \tag{4b}$$

$$\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}=j_s \tag{4c}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_s}{\partial s}=0 \tag{4d}$$

$$\frac{\partial e_y}{\partial x}-\frac{\partial e_x}{\partial y}=0 \tag{4e}$$

$$\frac{\partial e_s}{\partial y}-\frac{\partial e_y}{\partial s}=0 \tag{4f}$$

$$\frac{\partial e_x}{\partial s}-\frac{\partial e_s}{\partial x}=0 \tag{4g}$$

$$\frac{\partial e_x}{\partial x}+\frac{\partial e_y}{\partial y}+\frac{\partial e_s}{\partial s}=0 \tag{4h}$$

$$\frac{C_v^* T_0^*}{U_0^{*2}}\rho\left\{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+(1+w)\frac{\partial T}{\partial s}\right\}=\frac{p}{\rho}\left\{u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+(1+w)\frac{\partial \rho}{\partial s}\right\}+\frac{S}{R_m}\left\{j_x^2+j_y^2+j_s^2\right\} \tag{5}$$

where C_v^* denotes the specific heat of the fluid.

When the conductivity σ^* of the fluid is low, the coupling between the velocity and the magnetic field is small. Actually the uniform applied magnetic field is scarcely distorted for a very small conductivity. As the conductivity increases, the coupling becomes strong. It is well known that the magnetic field is frozen into the material in the extreme case of infinite conductivity. From these considerations, we can suppose that components of perturbation magnetic field h_x , h_y , h_s are the order of magnitude of the thickness ratio τ_0 of the body, at most, for a not so large conductivity, such that

for $Q \leqq 1$ and $R_m \leqq 1$

$$O(h_x)\leq O(\tau_0)\quad O(h_y)\leq O(\tau_0)\quad O(h_s)\leq O(\tau_0) \tag{6a}$$

On the other hand, we can estimate the order of magnitude of the components of the perturbation velocities and the density near the surface of the body based on the knowledge of the conventional flow past a slender body without a magnetic field (13) such that

$$O(u) = O(\tau_1) \qquad O(v) = O(\tau_1) \qquad O(w) = O\left(\tau_1^2 \log \frac{1}{\tau_1}\right)$$
$$O(\rho) = 1 + O\left(\tau_1^2 \log \frac{1}{\tau_1}\right) \qquad \tau_1 \ll 1 \tag{6b}$$

Remembering these order estimations, and retaining the leading terms, from Equations (3a), (3b) and (3c) we obtain

$$j_x = R_m(v - h_y + e_x) \tag{7a}$$

$$j_y = R_m(h_x - u + e_y) \tag{7b}$$

$$j_s = R_m(u h_y - v h_x + e_s) \tag{7c}$$

Since every term in each equation may be considered to be the same order of magnitude, we have

$$O(e_x) = O(\tau_1) \qquad O(e_y) = O(\tau_1) \qquad O(e_s) = O(\tau_1^2)$$
$$O(j_x) = O(R_m\tau_1) \qquad O(j_y) = O(R_m\tau_1) \qquad O(j_s) = O(R_m\tau_1^2) \tag{6c}$$

We shall take a line element $d\ell$ along a stream line defined by

$$\frac{u}{q} = \frac{dx}{d\ell} \qquad \frac{v}{q} = \frac{dy}{d\ell} \qquad \frac{1+w}{q} = \frac{ds}{d\ell}$$

where q denotes the magnitude of the nondimensional velocity. Integrating Equations (1a)-(1c) with respect to ℓ along a stream line, and remembering the order of estimations (6a)-(6c) we shall obtain

$$c_p = -2w - (u^2 - v^2) - 2Q\int_0^{\ell}\{(v-h_y)(v-h_y+e_x) + (h_x-u)(h_x-u+e_y)\}d\ell \tag{8a}$$
$$+ O(Q\tau_1^3) + O\left(\tau_1^4 \log^2 \frac{1}{\tau_1}\right)$$

where c_p denotes the pressure coefficient defined by

$$c_p = \frac{p^* - p_0^*}{\frac{1}{2}\rho_0^* U_0^{*2}} = 2(p - p_0)$$

Since the body is very thin, and stream lines are almost parallel to the uniform flow velocity, we can approximately replace the variable ℓ by S. This is the quadratic formula for the pressure coefficient. If we put $Q = 0$ in Equation (8a) we have the pressure formula for the conventional case (14). By the use of Equation (8a) we can calculate the pressure distribution, if we find the velocity and the magnetic field over the body.

Especially in the axially symmetric case, the induced electric field can be shown to be zero because of the geometrical symmetry.(9) Thus Equation (8a) is reduced to

$$c_p = -2w - (u^2 + v^2) - \frac{2S}{R_m}\int_0^s (j_x^2 + j_y^2)\,ds \tag{8b}$$

The last term of the right hand side of this equation represents the energy loss due to Joule heat. The pressure drop is connected with the irreversible process.

II LINEARIZATION OF EQUATIONS FOR FLUID

We shall linearize the fundamental equations for the fluid flow obtained in the last section. If we neglect the terms involving the square of perturbation quantities, the equations of motion and the equation of continuity are reduced to

$$\frac{\partial u}{\partial s} + \frac{\partial p}{\partial x} = Q\left\{e_y + h_x - u\right\} \tag{9a}$$

$$\frac{\partial v}{\partial s} + \frac{\partial p}{\partial y} = Q\left\{-e_x + h_y - v\right\} \tag{9b}$$

$$\frac{\partial w}{\partial s} + \frac{\partial p}{\partial s} = 0 \tag{9c}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial s} + \frac{\partial \rho}{\partial s} = 0 \tag{10}$$

Combining Equation (4a) with Equation (7a), we obtain

$$\frac{\partial h_s}{\partial y} - \frac{\partial h_y}{\partial s} = R_m\left\{e_x + v - h_y\right\} \tag{11a}$$

Similarly, from Equations (4b) and (7b), we obtain

$$\frac{\partial h_x}{\partial s} - \frac{\partial h_s}{\partial x} = R_m\left\{e_y + h_x - u\right\} \tag{11b}$$

Considering the order estimations (6c), we shall find that the axial components of the electric field e_s and the electric current density j_s can be safely neglected in comparison with the x- and the y components within the accuracy of the linearized theory. Thus (4c) and (4h) can be respectively, approximated by

$$\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = 0 \quad (11c) \qquad \frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} = 0 \quad (11d)$$

Equations (4f) and (4g) imply that $\partial e_x/\partial s$ and $\partial e_y/\partial s$ vanish in the present approximation. Thus the induced electric field forms a quasi-two-dimensional field. Remembering Equations (6c), we shall find that the last parenthesis of the right hand side of Equation (5) is the order of $o(Q\tau_1^2)$. Neglecting the higher order terms, we have

$$\frac{C_v^* T_0^*}{U_0^{*2}} \rho \frac{\partial T}{\partial s} = \frac{p}{\rho} \frac{\partial \rho}{\partial s}$$

Combining this equation with the equation of state of an ideal gas, we have

$$\frac{p}{\rho^{\gamma^*}} = \text{const}$$

where γ^* denotes the ratio of the specific heats c_p^*/c_v^*. The fluid conforms the law of the adiabatic change within the accuracy of the linearized theory.

Introducing the Mach number of the uniform flow

$$M_0 = U_0^*/a_0^*$$

where a_0^* is the free stream speed of sound, we have

$$M_0^2 = \left(\frac{dp}{d\rho}\right)_0 \tag{12}$$

From Equation (11c) we shall introduce the quasi-two-dimensional magnetic potential such as

$$h_x = \frac{\partial \psi}{\partial x} \qquad h_y = \frac{\partial \psi}{\partial y} \tag{13a}$$

Eliminating h_s from Equations (11a) and (11b), and considering Equations (11c) and (11d), we obtain

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

From this equation, we can introduce the quasi-two-dimensional velocity potential defined by

$$u = \frac{\partial \varphi}{\partial x} \qquad v = \frac{\partial \varphi}{\partial y} \tag{13b}$$

From Equation (11d), we shall introduce the electric force function Ω such as

$$e_x = \frac{\partial \Omega}{\partial y} \qquad e_y = -\frac{\partial \Omega}{\partial x} \tag{13c}$$

Combining these equations described with each other, we shall obtain the fundamental equations governing φ , ψ and Ω such that

$$\Delta\left[\left\{\Delta - B^2\frac{\partial^2}{\partial s^2} - B^2 Q\frac{\partial}{\partial s}\right\}\left\{\Delta + \frac{\partial^2}{\partial s^2} - R_m\frac{\partial}{\partial s}\right\} - B^2 Q R_m \frac{\partial^2}{\partial s^2}\right]\begin{bmatrix}\varphi \\ \psi\end{bmatrix} = 0 \tag{14}$$

$$\Delta \Omega = 0 \tag{15}$$

with $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $B^2 = M_\infty^2 - 1$

If the uniform flow is subsonic, B^2 takes the negative sign. Besides, the unknown functions φ , ψ and Ω have to satisfy the following supplementary equations.

$$\Delta\varphi - B^2\left[\frac{\partial^2\varphi}{\partial s^2} + Q\left(\frac{\partial\varphi}{\partial s} - \frac{\partial\psi}{\partial s} + \frac{\partial\Omega}{\partial s}\right)\right] = 0 \tag{16a}$$

$$\Delta\psi + \left[\frac{\partial^2\psi}{\partial s^2} + R_m\left(\frac{\partial\varphi}{\partial s} - \frac{\partial\psi}{\partial s} + \frac{\partial\Omega}{\partial s}\right)\right] = 0 \tag{16b}$$

$$\left(\Delta - B^2\frac{\partial^2}{\partial s^2} - B^2 Q\frac{\partial}{\partial s}\right)\Delta\varphi + B^2 Q\frac{\partial}{\partial s}\Delta\psi = 0 \tag{16c}$$

$$\left(\Delta + \frac{\partial^2}{\partial s^2} - R_m\frac{\partial}{\partial s}\right)\Delta\psi + R_m\frac{\partial}{\partial s}\Delta\varphi = 0 \tag{16d}$$

The axial components of the perturbation velocity and the magnetic fields can be given by

$$w = \frac{\partial\varphi}{\partial s} + Q(\varphi - \psi + \Omega) \tag{17a}$$

$$h_s = \frac{\partial\psi}{\partial s} + R_m(\varphi - \psi + \Omega) \tag{17b}$$

Considering Equations (13b) and (17a) we find that the velocity field is irrotational in the sectional plane perpendicular to the s-axis, but it is rotational in the meridian plane. The situation is the same for the magnetic field.

III FUNDAMENTAL EQUATIONS FOR INSIDE ELECTROMAGNETIC FIELD

In the interior of the body, the Ohm's law is simply represented by

$$j_{xi} = R_b e_{xi} \quad (18a) \qquad j_{yi} = R_b e_{yi} \quad (18b)$$

$$j_{si} = R_b e_{si} \quad (18c)$$

with $R_b = 4\pi \sigma_b^* U_0^* L_0^*$

where σ_b^* denotes the electric conductivity of the body, and the subscript "i" denotes the quantities in the interior of the body. The Maxwell's equations are represented by Equations (4a)-(4h). As described above (c.f. equation (6c)), the axial component of the outside electric field e_s is negligibly small. If we consider that the s-component of the electric field should be continuous on the surface of the body, and the thickness of the body is the order of $O(\tau_1)$, we can conclude the s-component e_{si} of the inside electric field to be the order of $O(\tau_1^2)$. Remembering Equation (18c) we can safely neglect e_{si} and j_{si} within the accuracy of the linearized theory. Considering Equations (4c) and (4h), and neglecting the higher order terms, we can introduce the inside magnetic potential ψ_i and the inside electric force function Ω_i defined by

$$h_{xi} = \frac{\partial \psi_i}{\partial x} \qquad h_{yi} = \frac{\partial \psi_i}{\partial y} \quad (19a)$$

$$e_{xi} = \frac{\partial \Omega_i}{\partial y} \qquad e_{yi} = -\frac{\partial \Omega_i}{\partial x} \quad (19b)$$

Using the similar treatment as that for the outer field, we shall obtain the following fundamental and supplementary equations governing ψ_i and Ω_i,

$$\Delta\left(\Delta + \frac{\partial^2}{\partial s^2}\right)\psi_i = 0 \quad (20)$$

$$\Delta \Omega_i = 0 \quad (21)$$

$$\Delta \psi_i + \frac{\partial^2 \psi_i}{\partial s^2} + R_b \frac{\partial \Omega_i}{\partial s} = 0 \quad (22)$$

The axial component h_{si} of the magnetic field can be given by

$$h_{si} = \frac{\partial \psi_i}{\partial s} + R_b \Omega_i \quad (23)$$

IV GENERAL SOLUTIONS OF FUNDAMENTAL EQUATIONS

(A) OUTER SOLUTIONS

First we shall consider the supersonic case. Since the fundamental equation (14) is linear, the general solution should be represented by the sum of three independent solutions φ_g, φ_a and φ_b such that

$$\varphi = \varphi_g + \varphi_a + \varphi_b \quad (24)$$

The solution φ_g satisfies the Laplace's equation,

$$\Delta \varphi_g = 0 \quad (25)$$

Introducing the Laplace transform $\overline{\varphi}_a$ of the solution φ_a, and the Fourier transform $\overline{\varphi}_b$ of the solution φ_b with respect to s, we shall obtain the equations governing $\overline{\varphi}_a$ and $\overline{\varphi}_b$ such that

$$(\Delta-\alpha)\overline{\varphi}_a=0 \tag{26a}$$

$$(\Delta-\beta)\overline{\varphi}_b=0 \tag{26b}$$

with

$$\alpha=\frac{\lambda}{2}\Big\{(B^2-1)\lambda+(B^2Q+R_m) + \sqrt{(B^2+1)^2\lambda^2+2(B^2+1)(B^2Q-R_m)\lambda+(B^2Q+R_m)^2} \tag{27a}$$

$$\beta=\frac{\omega}{2}\Big\{\omega(1-B^2)-i(B^2Q+R_m) + \sqrt{(B^2+1)^2\omega^2+2i\omega(B^2+1)(B^2Q-R_m)-(B^2Q+R_m)^2} \tag{27b}$$

where λ and ω denote the parameters of the Laplace and the Fourier transforms, respectively. The solution of Equation (26a) satisfying the condition that the perturbation velocity should vanish at infinity, can be expressed such as

$$\overline{\varphi}_a(r,\theta,\lambda)=\sum_{n=0}^{\infty}A_n(\lambda)K_n(\sqrt{\alpha}\,r)\cos(n\theta+\alpha_n(\lambda)) \tag{28}$$

where r, θ, s denote the cylindrical coordinates, $A_n(\lambda)$ and $\alpha_n(\lambda)$ the arbitrary functions of λ, and $K_n(\sqrt{\alpha}\,r)$ is the n-th order modified Bessel function of the second kind. Near the body, where r is small, the Bessel functions can be expanded into the following series,

$$K_0(\sqrt{\alpha}\,r)=-\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}\right\}-\tfrac{1}{4}\alpha r^2\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}-1\right\} - \tfrac{1}{64}\alpha^2r^4\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}-\tfrac{3}{2}\right\}+\cdots\cdots$$

$$K_1(\sqrt{\alpha}\,r)=\frac{1}{\sqrt{\alpha}\,r}+\tfrac{1}{2}\sqrt{\alpha}\,r\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}-\tfrac{1}{2}\right\} + \tfrac{1}{16}(\sqrt{\alpha}\,r)^3\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}-\tfrac{5}{4}\right\}+\cdots\cdots$$

$$K_2(\sqrt{\alpha}\,r)=\tfrac{1}{2}\left\{\frac{4}{\alpha r^2}-1\right\}-\frac{\alpha r^2}{8}\left\{\log\left(\tfrac{1}{2}\sqrt{\alpha}\,r\right)+\tilde{C}-\tfrac{3}{4}\right\}+\cdots\cdots$$

$$K_n(\sqrt{\alpha}\,r)=\tfrac{1}{2}\left[(n-1)!\left(\frac{2}{\sqrt{\alpha}\,r}\right)^n-(n-2)!\left(\frac{2}{\sqrt{\alpha}\,r}\right)^{n-2}+\frac{(n-3)!}{2}\left(\frac{2}{\sqrt{\alpha}\,r}\right)^{n-4}\right] \quad \text{for } n\geq 3$$

with $\tilde{C}=0.5772$

Remembering (27a), we shall find that α takes a moderate value for a moderate value of λ, when the parameters B^2, Q and R_m are the order of unity, $O(1)$ In such a case we may take the first three terms as an approximation of the Bessel functions for a small value of r. Moreover, the second and the third terms in the series expansion of K_0 are much smaller than the first term. Since the second and the third terms are not essential, they may be safely replaced by

$$-\frac{1}{4}\alpha r^2(\log r-1) \qquad \text{and} \qquad -\frac{1}{64}\alpha^2 r^4\left(\log r-\frac{3}{2}\right)$$

From similar considerations, K_1 and K_2 can be approximately represented by

$$K_1(\sqrt{\alpha}\, r) = \frac{1}{\sqrt{\alpha}\, r}+\frac{\sqrt{\alpha}\, r}{2}\log r+\frac{1}{16}(\sqrt{\alpha}\, r)^3\left(\log r-\frac{3}{4}\right)$$

$$K_2(\sqrt{\alpha}\, r) = \frac{1}{2}\left(\frac{4}{\alpha r^2}-1\right)-\frac{\alpha}{8}r^2\log r$$

For convenience, we shall introduce the complex variable in the sectional plane such as

$$z = x+iy = re^{i\theta}$$

Using the approximate expressions of K_n, and applying the inverse Laplace transformation to $\bar{\varphi}_a$, we obtain the solution in the following form,

$$\varphi_a = \varphi_{a_0}+\varphi_{a_1}+\varphi_{a_2}+\varphi_{a_3}$$

with

$$\begin{aligned}
\varphi_{a_0} &= \left\{\log\frac{1}{2}\sqrt{\alpha}+\tilde{C}\right\}a_0\\
\varphi_{a_1} &= R_e\left[a_0\log z+\sum_{n=1}^{\infty}a_n z^{-n}\right]\\
\varphi_{a_2} &= R_e\,\frac{\alpha}{4}\,\bar{z}\left[a_0 z(\log z-1)+a_1\log z\bar{z}-\sum_{n=2}^{\infty}\frac{a_n}{n-1}z^{-n-1}\right]\\
\varphi_{a_3} &= R_e\,\frac{\alpha^2}{32}\,\bar{z}^2\left[\frac{a_0}{2}z^2\left(\log z-\frac{3}{2}\right)+a_1 z\left\{\log z\bar{z}-\frac{3}{2}\right\}\right.\\
&\qquad\left.-a_2\log z\bar{z}+\sum_{n=3}^{\infty}\frac{a_n}{(n-1)(n-2)}z^{-(n-2)}\right]
\end{aligned} \tag{29a}$$

$$\bar{z} = re^{-i\theta}$$

where $a_0(s)$ and $a_n(s)$ denote arbitrary pure real and complex functions of s, R_e denotes the real part of complex function, and α means the linear operator corresponding to the expression (27a). Similarly, by the use of the Fourier transformation, we can obtain the solution φ_b in the following form

$$\varphi_b = \varphi_{b_0}+\varphi_{b_1}+\varphi_{b_2}+\varphi_{b_3} \tag{29b}$$

The functions $\varphi_{b_0}, \varphi_{b_1}$, φ_{b_2}, and φ_{b_3} are obtained by replacing α, a_0, and a_n in Equation (29a) by β, b_0 and b_n, where β means the linear operator corresponding to the expression (27b), and b_0 and b_n denote the arbitrary pure real and the complex functions of s different from $a_0(s)$ and $a_n(s)$. The general solution φ_g of Equation (25) satisfying the condition at infinity should be given by

$$\varphi_g = R_e\left[-i\Gamma_1\log z+\sum_{n=1}^{\infty}g_n z^{-n}\right] \tag{30}$$

where $\Gamma_1(s)$ and $g_n(s)$ denote the arbitrary pure real and the complex functions of s, respectively. Substituting Equations (29a), (29b) and (30) into Equation (24), we shall obtain the solution such that

$$\varphi = \varphi_0+\varphi_1+\varphi_2+\varphi_3$$

with

$$\varphi_0 = \varphi_{a_0} + \varphi_{b_0} \qquad \varphi_2 = \varphi_{a_2} + \varphi_{b_2}$$

$$\varphi_1 = \varphi_g + \varphi_{a_1} + \varphi_{b_1} \qquad \varphi_3 = \varphi_{a_3} + \varphi_{b_3} \tag{31a}$$

Considering Equation (29a), we can find the following relations,

$$\Delta\varphi_g = \Delta\varphi_{a_1} = \Delta\varphi_{b_1} = 0 \qquad \Delta\varphi_1 = 0$$

$$\Delta\varphi_{a_2} = \alpha\varphi_{a_1} \quad \Delta\varphi_{b_2} = \beta\varphi_{b_1} \qquad \Delta\Delta\varphi_2 = 0 \tag{31b}$$

$$\Delta\varphi_{a_3} = \alpha\varphi_{a_2} \quad \Delta\varphi_{b_3} = \beta\varphi_{b_2} \qquad \Delta\Delta\Delta\varphi_3 = 0$$

and

$$O(\varphi_1) = O(\log \tfrac{1}{\tau_1})\,O(\varphi_0)$$

$$O(\varphi_2) = O(\tau_1^2)\,O(\varphi_1) \tag{31c}$$

$$O(\varphi_3) = O(\tau_1^2)\,O(\varphi_2)$$

Since the fundamental equation governing ψ is the same as that governing φ, we can obtain the solution ψ in the similar form,

$$\psi = \psi_0 + \psi_1 + \psi_2 + \psi_3 \qquad \psi_2 = \psi_{c_2} + \psi_{d_2}$$

$$\psi_0 = \psi_{c_0} + \psi_{d_0} \tag{32}$$

$$\psi_1 = \psi_h + \psi_{c_1} + \psi_{d_1} \qquad \psi_3 = \psi_{c_3} + \psi_{d_3}$$

The functions ψ_h, ψ_{c_0}, ψ_{d_0}, ψ_{c_1}, ψ_{d_1}, ψ_{c_2}, ψ_{d_2}, ψ_{c_3}, and ψ_{d_3} can be obtained by replacing the arbitrary functions Γ_1, a_0, $\mathbb{a}_n$, b_0, and $\mathbb{b}_n$ in the solution φ with the different arbitrary functions Γ_2, c_0, $\mathbb{c}_n$, d_0, and $\mathbb{d}_n$, respectively. We have the similar relations to (31b) and (31c).

The solution of Equation (15) satisfying the condition at infinity can be readily written in the following form,

$$\Omega = R_e\left[-i\Gamma_3 \log z + \sum_{n=1}^{\infty} \mathbb{K}_n z^{-n}\right] \tag{33}$$

Next, in the subsonic case, both the independent solutions φ_a and φ_b are obtained by the Fourier transformation. The results are given in the same expression as those of the supersonic case by replacing the linear operators α and β with γ and δ.

(B) INNER SOLUTIONS

The solution of Equation (20) without any singularity along the axis ($r=0$) except the logarithmic singularity is given by

$$\psi_i = \psi_m + \psi_f \tag{34}$$

The solution ψ_m is the harmonic function such as

$$\psi_m = R_e\left[-i\Gamma_4 \log z + \sum_{n=1}^{\infty} m_n z^n\right] \tag{35}$$

The Fourier transform $\overline{\psi}_f$ of the function ψ_f with respect to S should satisfy the equation

$$(\Delta - \omega^2)\,\overline{\psi}_f = 0 \tag{36}$$

The general solution of Equation (36) without any singularity along the axis is obtained in the following form

$$\bar{\Psi}_f = \sum_{n=0}^{\infty} F_n(\omega) J_n(i\omega r) \cos\left(n\theta) + \eta_n(\omega)\right) \tag{37}$$

where $F_n(\omega)$ and $\eta_n(\omega)$ are the arbitrary functions of ω, and $J_n(i\omega r)$ denotes the n-th order Bessel function of the first kind. Since the thickness of the body is very small, the Bessel functions can be approximated by the first two-terms of their expansions into the r-power series, such that

$$J_n(i\omega r) = \frac{i^n \omega^n r^2}{n!\, 2^n} + \frac{i^n \omega^{n+2} r^{n+2}}{(n+1)!\, 2^{n+2}}$$

Using this approximation, and applying the inverse Fourier transformation to $\bar{\Psi}_f$, we shall obtain the solution Ψ_i such that

$$\Psi_i = \Psi_{f_0} + \Psi_{i_1} + \Psi_{f_2}$$

with

$$\Psi_{i_1} = \Psi_m + \Psi_{f_1} \qquad \Psi_{f_1} = \mathcal{R}_e \sum_{n=1}^{\infty} f_n z^n$$

$$\Psi_{f_0} = f_0(s) - \frac{r^2}{4} f_0''(s) \qquad \Psi_{f_2} = -\mathcal{R}_e \frac{z\bar{z}}{4} \sum_{n=1}^{\infty} \frac{f_n''(s)}{(n+1)} z^n \tag{38a}$$

Considering the solution (38a), we shall find the following relations,

$$\Delta\Psi_{i_1} = 0 \qquad \Delta\Psi_{f_2} = -\frac{\partial^2}{\partial s^2}\Psi_{f_1} \qquad \Delta\Delta\Psi_{f_2} = 0 \tag{38b}$$

and

$$O(\Psi_{f_2}) = O(\tau_1^2)\, O(\Psi_{f_1}) \tag{38c}$$

The solution Ω_i of Equation (21) can be easily found such that

$$\Omega_i = \mathcal{R}_e\left[-i\Gamma_\sigma(s)\log z + \sum_{n=1}^{\infty} t_n z^n\right] \tag{39}$$

As we shall discuss later, the logarithmic singularity of the inside solutions is caused by the current flowing in an infinitesimally thin filament of infinite conductivity inserted along the axis of the body.

V SUPPLEMENTARY EQUATIONS

The general solutions obtained above contain several arbitrary functions of s. They should be determined by satisfying the boundary conditions and the supplementary equations (16a)-(16d) and (22).

Remembering Equations (31a)-(31c), and neglecting the higher order terms $o(\tau_1^2)$ we can write Equation (16a) such that

$$\alpha\varphi_{a_1} + \beta\varphi_{b_1} - B^2\left\{\frac{\partial^2\varphi}{\partial s^2} + Q\frac{\partial}{\partial s}(\varphi_1 - \Psi_1 + \Omega)\right\} = 0 \tag{40}$$

Substituting the expressions (29a) into Equation (40) and comparing the coefficients of the terms involving the same function of z, we shall obtain

$$\frac{d^2\Gamma_1}{ds^2} + Q\left\{\frac{d\Gamma_1}{ds} - \frac{d\Gamma_2}{ds} + \frac{d\Gamma_3}{ds}\right\} = 0 \tag{41a}$$

$$\alpha a_0 + \beta b_0 - B^2\left[a_0''(s) + b_0''(s) + Q\left\{a_0'(s) + b_0'(s) - c_0'(s) - d_0'(s)\right\}\right] = 0 \tag{41b}$$

and

$$\alpha\, a_n + \beta b_n - B^2 \left[a''_n(s) + b''_n(s) + g''_n(s) \right.$$

$$\left. + Q\left\{ a'_n(s) + b'_n(s) + g'_n(s) - c'_n(s) - d'_n(s) - h'_n(s) + k'_n(s) \right\} \right] = 0 \qquad \text{for } n \geq 1 \tag{41c}$$

Integrating Equation (41a), and considering the condition that all the perturbations should vanish at infinity upstream, we have

$$\frac{d\Gamma_1}{ds} + Q(\Gamma_1 - \Gamma_2 + \Gamma_3) = 0 \tag{41a'}$$

Similarly, from Equation (16b) we obtain

$$\frac{d\Gamma_2}{ds} + R_m(\Gamma_1 - \Gamma_2 + \Gamma_3) = 0 \tag{41d}$$

$$\alpha c_0 + \beta d_0 + \left[c''(s) + d''_0(s) + R_m \left\{ a'_0(s) + b'_0(s) - c'_0(s) - d'_0(s) \right\} \right] = 0 \tag{41e}$$

and $\alpha c_n + \beta d_n + \left[c''_n(s) + d''_n(s) + h''_n(s) + R_m \left\{ a'_n(s) + b'_n(s) \right.\right.$

$$\left.\left. + g'_n(s) - c'_n(s) - d'_n(s) - h'_n(s) + k'_n(s) \right\} \right] = 0 \qquad n \geq 1 \tag{41f}$$

By the use of the same procedure, Equations (16c) and (16d) are respectively, reduced to

$$\alpha^2 a_n + \beta^2 b_n - B^2 \left[\alpha a''_n(s) + \beta b''_n(s) + Q\left\{ \alpha a'_n + \beta b'_n - \alpha c'_n - \beta d'_n \right\} \right] = 0 \tag{41g}$$

$$\alpha^2 c_n + \beta^2 d_n + \left[\alpha c''_n(s) + \beta d''_n(s) + R_m \left\{ \alpha a'_n + \beta b'_n - \alpha c'_n - \beta d'_n \right\} \right] = 0 \tag{41h}$$

with $a_0 \equiv a_0 \qquad b_0 \equiv b_0 \qquad c_0 \equiv c_0 \qquad$ and $d_0 \equiv d_0$

The supplementary condition (22) for the inside field supplies

$$\frac{d\Gamma_4}{ds} + R_L \Gamma_5 = 0 \tag{41i}$$

$$\frac{dm_n}{ds} + R_L t_n = 0 \tag{41j}$$

VI BOUNDARY CONDITIONS

We shall take a plane perpendicular to the S -axis. The cross section of the body in the plane forms a closed curve, C, say. The outward directing normal vector and the counterclockwise tangential vector of the curve C are denoted by n and τ , respectively. Within the accuracy of the linearized theory, the boundary conditions can be given in the following form

(i) Since the fluid must flow past the body, we have $\dfrac{\partial \varphi_i}{\partial n} = \dfrac{\partial n}{\partial s}$ (42a)

If the body is given, the right hand side of this equation is a known function. Integrating Equation (42a) along the curve C with respect to τ, we obtain

$$a_0(s) + b_0(s) = \frac{1}{2\pi} Z'(s) \tag{42b}$$

where $Z(s)$ denotes the area of the cross section of the body.

(ii) Since the magnetic field should be continuous on the surface of the surface of the body, we have

$$\frac{\partial \Psi_1}{\partial \tau} = \frac{\partial \Psi_{f_0}}{\partial \tau} + \frac{\partial \Psi_{i_1}}{\partial \tau} \tag{43a}$$

$$\frac{\partial \Psi_1}{\partial n} = \frac{\partial \Psi_{f_0}}{\partial n} + \frac{\partial \Psi_{i_1}}{\partial n} \tag{43b}$$

$$\frac{\partial \Psi_1}{\partial s} - R_m(\Psi_1 - \varphi_1 - \Omega) = \frac{\partial \Psi_{f_0}}{\partial s} + \frac{\partial \Psi_{i_1}}{\partial s} + R_b \Omega_1 \tag{43c}$$

Integrating Equations (43a) and (43b) along the curve C with respect to τ, we obtain, respectively

$$\Gamma_2 = \Gamma_4 \tag{43d}$$

and
$$C_0 + d_0 = -\frac{f_0''(s)}{2\pi} G(s) \tag{43e}$$

with $G(s) = \frac{1}{2} \oint_c r \sqrt{1 - \left(\frac{\partial r}{\partial \tau}\right)^2} \, d\tau$

When $\partial r/\partial \tau$ is much smaller than unity, $G(s)$ can be approximated by the sectional area $Z(s)$ of the body.

(iii) Since the tangential component of the electric field should be continuous on the surface of the body, we have

$$\frac{\partial \Omega}{\partial n} = \frac{\partial \Omega_i}{\partial n} \tag{44}$$

The total charge surface density σ_1^* induced on the surface of the body should be given by

$$\sigma_1 = \frac{\partial \Omega}{\partial \tau} - \frac{\partial \Omega_i}{\partial \tau} \tag{45a}$$

with $\sigma_1 = \frac{4\pi \sigma_1^*}{H_0^* U_0^*}$

Integrating Equation (45a) along the curve C with respect to τ, we shall obtain

$$\oint_c \sigma_1 \, d\tau = 2\pi (\Gamma_3 - \Gamma_5) \tag{45b}$$

The left-hand side of this equation denotes the amount of the surface charge per unit width in the s-direction.

The total amount q_1^* of the surface charge can be evaluated by

$$q_1 = \int_0^1 \oint_c \sigma_1 \, d\tau \, ds = 2\pi \int_0^1 \{\Gamma_3(s) - \Gamma_5(s)\} \, ds \tag{45c}$$

with $q_1 = \frac{4\pi q_1^*}{H_0^* U_0^* L_0^{*2}}$

(iv) We shall consider the condition at the axis ($r=0$).

(a) In the usual case we have no singularity within the body. All the variables should be continuous and finite in the interior of the body.

(b) We shall consider such a geometrical configuration that an infinitesimally thin filament with infinite conductivity is inserted along the s-axis. The electric current flows in the filament without any electric field along the filament. Moreover some electric charge can be induced along the filament, and the source singularity of the electric field at $r=0$ can be permitted. As a result, we may have a circular magnetic field and a circulation of the fluid.

Considering the continuity of current, we have

$$J = 2\pi \Gamma_2 \qquad \text{with} \qquad J = \frac{4\pi J^*}{H_0^* L_0^*} \tag{46}$$

where J^* denotes the electric current flowing in the filament. The length density ρ_1 of the electric charge induced along the filament can be obtained by

$$\rho_1 = \oint \frac{\partial \Omega_i}{\partial \tau} d\tau = 2\pi \Gamma_5 \tag{47}$$

Remembering the fact that Γ_5 and Γ_3 denote the source strengths of the inner and the outer radial electric fields, respectively, we can realize the physical meaning of Equations (45b) and (47). For example, we shall consider the case, where both Γ_5 and Γ_3 are positive. In the interior of the body lines of electric force corresponding to the strength Γ_5 starts from the filament and terminates at the surface of the body. In the outside of the body lines of electric force corresponding to the strength Γ_3 starts from the surface toward infinity.

VII CIRCULATION

We shall obtain the five relations among the functions Γ_1, Γ_2, Γ_3, Γ_4 and Γ_5 represented by Equations (41a'), (41d), (41i), (43d) and (46).

(a) If there is no filament along the axis, we can put

$$J(s) = 0 \tag{48}$$

Combining Equation (48) with the five relations described above, we shall obtain

$$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \Gamma_5 = 0 \tag{49}$$

Thus we have found that the circulation of the fluid vanishes together with the singularity at the axis.

(b) We shall consider the case, where the filament is inserted along the s-axis, and terminates at the both pointed ends of the body. In this case we can put

$$\begin{aligned} &J(s) = 0 \quad \text{for} \quad s \leqq 0 \quad \text{and} \quad 1 \leqq s \\ &J(s) \neq 0 \quad \text{for} \quad 0 < s < 1 \end{aligned} \tag{50}$$

where physically considering, $J(s)$ should be a continuous function of s. Combining Equation (50) with the five relations amoung Γ_1, Γ_2, Γ_3, Γ_4, and Γ_5, we obtain

$$\Gamma_2 = \frac{1}{S}\Gamma_1 \tag{51a}$$

$$\Gamma_3 = \left(\frac{1}{S} - 1\right)\Gamma_1 - \frac{1}{Q}\frac{d\Gamma_1}{ds} \tag{51b}$$

$$\Gamma_4 = \frac{1}{S}\Gamma_1 \tag{51c}$$

$$\Gamma_5 = -\frac{1}{Q_b}\frac{d\Gamma_1}{ds} \tag{51d}$$

$$J = \frac{2\pi}{S}\Gamma_1 \tag{51e}$$

with $Q = SR_m$, $Q_b = SR_b$

By the use of Equations (45c) and (47) we can calculate the total amount of the electric charge on the surface of the body, and the length density of the charge along the filament, respectively, such as

$$q_1 = 2\pi \int_0^1 \left\{ \left(\frac{1}{S} - 1\right)\Gamma_1 - \frac{1}{Q}\frac{d\Gamma_1}{ds} + \frac{1}{Q_b}\frac{d\Gamma_1}{ds} \right\} ds \tag{51f}$$

and

$$\rho_1 = -\frac{2\pi}{Q_b}\frac{d\Gamma_1}{ds} \tag{51g}$$

Thus, if the circulation Γ_1 of the flow is determined, all other quantities Γ_2, Γ_3, Γ_4, Γ_5, J, q_1, and ρ_1 can be obtained. It is of interest to consider the following extreme cases.

(i) If the conductivity σ_b^* of the body vanishes, all Γ_i except Γ_5 should vanish. The velocity field has no circulation as well as the magnetic field. No current flows in the filament. Remembering Equation (39), we realize that Γ_5 represents the radial electric field in the interior of the body. Then the inside radial electric field may remain in this case. As a result ρ_1 and q_1 may exist.

(ii) In the case, where the conductivity σ^* of the fluid vanishes, we find that all Γ_s except Γ_3 vanish. The velocity and the magnetic fields have no circulation. This case corresponds to the conventional case. The outside radial electric field corresponding to Γ_3 may remain.

(iii) In a hypersonic flow, in the presence of an applied magnetic field, we use the magnetohydrodynamic Stokes approximation frequently. In this approximation we suppose that R_m and R_b vanish, but Q and Q_b still remain finite, because S becomes infinite. The circulations Γ_2 and Γ_4 of the outer and the inner magnetic fields vanish, but the circulation Γ_1 of the flow and the electric radial fields Γ_3 and Γ_5 may remain. Corresponding to this fact, the current J vanishes, while the induced charge distributions q_1 and ρ_1 remain.

We shall return to the general case, where R_m and Q are the order of unity at most. Remembering Equations (31b) and (42a), we realize that φ_1 should satisfy the two-dimensional Laplace's equation and the Neuman's boundary condition. It is well known that the solution φ_1 can be determined except an arbitrary function $R_e\{-i\Gamma_1 \log z\}$. If the curve C is smooth, we have no mathematical reason to determine the value of $\Gamma_1(s)$. If the curve C has a sharp edge, $\partial\varphi_1/\partial r$ may become infinite at the edge. We can choose an appropriate value of Γ_1 in order to prevent the infinite velocity. These circumstances are similar to the conventional stationary flow past a two-dimensional obstacle with a sharp trailing edge. If the fluid could flow around the sharp corner in the initial stage of the magnetohydramic flow, the tangential velocity would take a very large value. Then the normal component of the induced current might become tremendously large, unless the body is an insulator. According to the interaction between the induced current and the applied uniform magnetic field, a large Lorentz force exerts on the fluid. Since the fluid is detained by the Lorentz force, it cannot flow around the sharp edge. Thus there occurs an appropriate circulation around the curve C which will make the position at the edge coincide with a stagnation point.

By the use of the Fourier theorem, we can obtain the following result. The outside region of the curve C in the z-plane can be conformally mapped into the outside region of the unit circle $e^{i\theta}$ in the ζ -plane by an appropriate analytic function $\zeta(z)$. The function dn/ds in Equation (42a) is expressed as the function of θ and s such that

$$\frac{dn}{ds} = E(\theta, s) \tag{52a}$$

If the functions $\zeta(z)$ and $E(\theta, s)$ are found, the circulation Γ_1 of the flow can be obtained such that

$$\Gamma_1 = \frac{1}{\pi}\sum_{n=1}^{\infty}\int_{-\pi}^{\pi} V(\theta, s)\sin n\theta \, d\theta \tag{52b}$$

with $V(\theta, s) = E(\theta, s)\left|\frac{dz}{d\zeta}\right|_{\eta=1}$

and $\zeta = \eta e^{i\theta}$

For an example, we shall take a slender body, the sectional form of which is a thin symmetrical Joukowski aerofoil shown in Fig. 1. The chord length and the thickness of the Joukowski profile are represented by the functions $c(s)$ and $d(s)$ of s, respectively. We shall suppose that $d(s)/c(s)$ is a constant, and that the symmetrical plane of the body has a small angle of attack, μ, against the uniform flow (c.f. Fig. 2). Applying the general expression (52b) to this problem, we obtain

$$\Gamma_1 = \frac{-\mu c(s)}{2}\left\{1 + \frac{4}{3\sqrt{3}}\left(\frac{d}{c}\right)\right\} \tag{53}$$

Since both the edges of the body are supposed to be pointed, $c(s)$ vanishes at $s=0$ and $s=1$. Remembering Equation (51e), we find that the assumption (50) is self-consistent in this example.

VIII EXAMPLES

Hereafter we shall restrict our discussion to the case of small value of R_m and Q. Expanding the right hand side of Equation (27a) into the R_m and Q power series, and neglecting the order of square of R_m and Q, we have

$$\alpha = B^2(\lambda^2 + Q\lambda) + O(R_m^2, Q^2)$$

Considering the character of the Laplace transform, we shall obtain the linear operators α and $\{\log \frac{1}{2}\sqrt{\alpha} + \tilde{C}\}$ in Equation (29a) such that,

$$\alpha = B^2\left(\frac{d^2}{ds^2} + Q\frac{d}{ds}\right) + O(Q^2, R_m^2) \tag{54a}$$

$$\begin{aligned}(\log \tfrac{1}{2}\sqrt{\alpha} + \tilde{C})a_0(s) = {} & a_0(s)\log\frac{B}{2} - \int_0^s a_0'(\xi)\log(s-\xi)\,d\xi \\ & + \frac{Q}{2}\int_0^s a_0(\xi)\,d\xi + O(Q^2, R_m^2)\end{aligned} \tag{54b}$$

Considering the character of the Fourier transform, we shall obtain the operators β and $\{\log \frac{1}{2}\sqrt{\beta} + \tilde{C}\}$ such that,

$$\beta = -\frac{d^2}{ds^2} + R_m \frac{d}{ds} \tag{54c}$$

$$\left(\log\frac{1}{2}\sqrt{\beta} + \tilde{C}\right) b_0(s) = b_0(s)\log\frac{1}{2} - \frac{1}{2}\int_0^s b_0'(\xi)\log(s-\xi)\,d\xi + \frac{1}{2}\int_s^1 b_0'(\xi)\log(\xi - s)\,d\xi - \frac{R_m}{2}\int_0^s b_0(\xi)\,d\xi \tag{54d}$$

In the subsonic case, both the operators γ and δ are connected with the Fourier transformation. We shall obtain

$$\gamma = -b^2\left(\frac{d^2}{ds^2} + Q\,\frac{d}{ds}\right) \tag{55a}$$

$$\left(\log\frac{1}{2}\sqrt{\gamma} + \tilde{C}\right) a_0(s) = a_0(s)\log\frac{b}{2} - \frac{1}{2}\int_0^s a_0'(\xi)\log(s-\xi)\,d\xi + \frac{1}{2}\int_s^1 a_0'(\xi)\log(\xi-s)\,d\xi + \frac{Q}{2}\int_0^s a_0(\xi)\,d\xi \tag{55b}$$

$$\delta \equiv \beta \tag{55c}$$

$$\left(\log\frac{1}{2}\sqrt{\delta} + \tilde{C}\right) \equiv \left(\log\frac{1}{2}\sqrt{\beta} + \tilde{C}\right) \tag{55d}$$

with $b^2 = -B^2 = 1 - M_0^2$

From the character of the Fourier transformation, $b_0'(s)$ should be finite for $-\infty \leq s \leq +\infty$. Considering Equation (42b), this requirement implies that $Z'(s)$ should be continuous for $-\infty \leq s \leq +\infty$. Then $Z'(s)$ should vanish at $s=0$ and $s=1$. Therefore the body should have a pointed nose, and a pointed or cylindrical tail.

We shall consider two examples.

(i) Slender body of an elliptic cross section at zero incidence (c.f. Fig.3). We shall suppose that no singularity is at the axis. The sectional form of the body is given by

$$\frac{x^2}{(K+\varepsilon)^2} + \frac{y^2}{(K-\varepsilon)^2} = 1$$

where $K(s)$ and $\varepsilon(s)$ are the functions of s, and ε/K is supposed to be a small constant. Neglecting the terms of the order of $O(\varepsilon^2/K^2)$, we can write the boundary conditions in the following form.

From Equations (42a) and (42b) we have

$$a_0(s) + b_0(s) = \frac{1}{2}\frac{d}{ds}(K^2) \tag{56a}$$

$$a_2(s) + b_2(s) + g_2(s) = -\frac{K^2}{2}(K\varepsilon' + K'\varepsilon) \tag{56b}$$

From Equation (43a), we have

$$\begin{aligned} &-2(c_0+d_0)\frac{\varepsilon}{K} - (c_2+d_2+h_2)\frac{2}{K^2} \\ &= -2(f_2+m_2)K^2 + f_0''(s)\,K\varepsilon \end{aligned} \tag{56c}$$

From Equations (43b) and (43e) we have

$$c_0 + d_0 = -\frac{K^2}{2} f_0''(s) \tag{56d}$$

$$-2(c_0+d_0)\frac{\varepsilon}{K} - (c_2+d_2+h_2)\frac{2}{K^2} = 2(f_2+m_2)K^2 \tag{56e}$$

From Equation (43c) we have

$$f_0'(s) = (c_0'(s) + d_0'(s))\log K - R_m\{(c_0+d_0) - (a_0+b_0)\}\log K \tag{56f}$$

$$\begin{aligned} &\{f_2'(s) + m_2'(s) + R_b t_2\}K^2 - \frac{1}{2}f_0'''(s)\,K\varepsilon \\ &= \frac{\varepsilon}{K}\{c_0'(s)+d_0'(s)\} + \frac{1}{K^2}\{c_2'(s)+d_2'(s)+h_2'(s)\} \\ &\quad - R_m\Big[\frac{\varepsilon}{K}\{c_0+d_0-(a_0+b_0)\} \\ &\quad + \frac{1}{K^2}\{c_2+d_2+h_2-(a_2+b_2+g_2)-k_2\}\Big] \end{aligned} \tag{56g}$$

From Equation (44) we have

$$t_2 = -\frac{k_2}{K^4} \tag{56h}$$

In this case all the complex functions $\mathfrak{a}_2(s)$, $\mathfrak{b}_2(s)$, etc. are reduced to the pure real functions $a_2(s)$, $b_2(s)$ etc., respectively. Combining these eight differential equations (56a)-(56h) with the seven differential equations (41b), (41c), (41e), (41f), (41g), (41h) and (41j), we can determine the fifteen arbitrary functions a_0, b_0, c_0, d_0, f_0, a_2, b_2, c_2, d_2, f_2, g_2, h_2, k_2, m_2, and t_2.

Thus we obtain the solutions φ , ψ , ψ_i , Ω , and Ω_i. Remembering Equation (31c) and neglecting the higher order terms, we have

$$\varphi = \varphi_0 + \varphi_1$$

where $\varphi_0 = \left(\log \frac{1}{2}\sqrt{\alpha} + \tilde{C}\right)\frac{1}{2} Z'(s)$ for the supersonic case

(57a)

$= \left(\log \frac{1}{2}\sqrt{\gamma} + \tilde{C}\right)\frac{1}{2} Z'(s)$ for the subsonic case

$$\varphi_1 = R_e\left\{\frac{1}{2\pi} Z'(s) \log z - K^3 \mathcal{E}'(s) \frac{1}{z^2}\right\}$$

where $\psi = \psi_0 + \psi_1$

$$\psi_0 = -\left(\log \tfrac{1}{2}\sqrt{\alpha} + \tilde{C}\right)\frac{R_m Z(s)}{2\pi(1+B^2)} + \left(\log \tfrac{1}{2}\sqrt{\beta} + \tilde{C}\right)\left\{\frac{R_m Z(s)}{2\pi(1+B^2)} - \frac{K^2}{2}\left(\frac{dh_{si}}{ds}\right)_{r=0}\right\}$$

(57b)

$\psi_0 = \alpha$ and β are replaced by γ and δ for the supersonic case / for the subsonic case

$$\psi_1 = R_e\left[-\frac{K^2}{2}\left(\frac{dh_{si}}{ds}\right)_{r=0} \log z - K^3 \mathcal{E}'(s) \frac{1}{z^2}\right]$$

with

$$\left(\frac{dh_{si}}{ds}\right)_{r=0} = \frac{R_m}{2\pi} \frac{d}{ds}\left\{Z'(s) \log K\right\}$$

where $\psi_i = f_0 + \psi_{i_1}$

$$f_0 = \frac{R_m}{2\pi}\int_0^s Z'(\xi) \log K(\xi)\, d\xi$$

$$\psi_{i_1} = \frac{\mathcal{E}}{4K}\left(\frac{dh_{si}}{ds}\right)_{r=0} R_e\left(z^2\right) \tag{57c}$$

$$\Omega = R_e\left[-\frac{K^3 \mathcal{E}}{4} \frac{R_m}{R_b + R_m}\left(\frac{dh_{si}}{ds}\right)_{r=0} \frac{1}{z^2}\right] \tag{57d}$$

$$\Omega_i = R_e\left[\frac{\mathcal{E}}{4K} \frac{R_m}{R_b + R_m}\left(\frac{dh_{si}}{ds}\right)_{r=0} z^2\right] \tag{57e}$$

The velocity field in the sectional plane consists of the diverging or converging radial flow and the quadrupole like flow. The perturbation magnetic field is proportional to R_m. The outer electric field is represented by the field due to a quadrupole at the origin. The lines of electric force for the inner field is similar to the stream lines of the fluid flowing along the inside corner of right angle. By the use of Equation (45a), the surface density σ_1 of induced charge is represented by

$$\sigma_1 = \varepsilon \left[\frac{dh_i}{ds}\right]_{r=0} \frac{R_m}{R_b + R_m} \sin 2\theta \tag{58}$$

The surface charge σ_1 takes the maximum value for an insulated body ($R_b = 0$). As the conductivity σ_b^* of the body increases, the current flowing through the body increases. As a result the charge σ_1 will decrease. Moreover this may cause the decrease of the electric field given by Equations (57d) and (57e). Remembering Equations (7a) and (7b), we can introduce the electric current function Φ defined by

$$j_x = \frac{\partial \Phi}{\partial y} \qquad\qquad j_y = -\frac{\partial \Phi}{\partial x} \tag{59}$$

where $\Phi = R_m (\varphi_1 - \psi_1 + \Omega)$

The outside current field is represented by superposing a quadrupole field on a circular field. The electric field is shown in Fig. 4 schematically.

(ii) Body of revolution at a small incidence. We shall consider the flow past a body of revolution at a small incidence, μ. As shown in Fig. 5, the x-axis is taken to be vertical. By the use of the similar treatment to the former example, we can solve the present case. Neglecting the order of square of μ, φ and ψ are approximately represented by $\varphi_0 + \varphi_1$ and $\psi_0 + \psi_1$, respectively. The functions φ_0 and ψ_0 are obtained by replacing $K(s)$ in Equations (57a) and (57b) by $R(s)$, where $R(s)$ denotes the radius of the circular cross section. The functions φ_1 and ψ_1 are obtained in the following form

$$\varphi_1 = R_e \left[\frac{1}{2\pi} Z'(s) \log z + \frac{1}{2\pi}\left(\mu s Z'(s) + 2\mu Z(s)\right)\frac{1}{z}\right] \tag{60a}$$

with

$$\psi_1 = R_e \left[-\frac{R^2(s)}{2}\left[\frac{dh_{si}}{ds}\right]_{r=0} \log z - \frac{R^2(s)}{2}\mu s \left[\frac{dh_{si}}{ds}\right]_{r=0} \frac{1}{z}\right]$$

$$\left[\frac{dh_{si}}{ds}\right]_{r=0} = \frac{R_m}{2\pi} \frac{d}{ds}\left\{Z'(s) \log R(s)\right\} \tag{60b}$$

The functions ψ_i, Ω, Ω_i and the surface charge distribution are obtained in the following form,

$$\Psi_i = f_o + \Psi_{i_1} \qquad (60c)$$

where $f_o = \frac{R_m}{2\pi}\int_0^s Z'(\xi)\log R(\xi)\,d\xi$

$$\Psi_{i_1} = -\frac{\mu s}{2}\left(\frac{d h_{si}}{d s}\right)_{r=0} R_e(z)$$

$$\Omega = -\frac{R_m}{R_m+R_b}\frac{\mu}{\pi} Z(s) R_e\left(\frac{1}{z}\right) \qquad (60d)$$

$$\Omega_i = \frac{R_m}{R_m+R_b}\mu R_e(z) \qquad (60e)$$

$$\sigma_i = \frac{2R_m}{R_m+R_b}\mu \sin\theta \qquad (61)$$

The velocity field in the sectional plane consists of the diverging or converging radial field and the dipole like field. The outside electric field is that due to a dipole at the origin, and the inside electric field is uniform. The coefficient R_m/R_m+R_b has the same physical meaning as in the previous example. The electric field and the surface charge distribution are schematically shown in Fig. 5.

IX TOTAL FORCE

The force exerted on a body in the uniform flow, and the applied magnetic field consists of the hydrodynamic pressure and the electro-magnetic force. Since there is no applied electrostatic field in the present problem, the total electrostatic force exerted on the body should vanish. As the Lorentz body force exerts on the induced current in the interior of the body by the applied magnetic field, it should vanish for an insulated body. According to the electromagnetics, the Lorentz body force can be expressed by the surface integral of the Maxwell stress tensor. By the use of the momentum theorem, the total force $\mathbf{F}^*$ can be expressed by the integral over a control surface, S_1 say, such that

$$\mathbf{F} = -\iint_{S_1} C_p \boldsymbol{\nu}\, dS_1 - 2\iint_{S_1} \rho \mathbf{v}(\mathbf{v}\cdot\boldsymbol{\nu})\, dS_1 + 2S\iint_{S_1}\left\{(\mathbf{H}\cdot\boldsymbol{\nu})\mathbf{H} - \frac{1}{2}H^2\boldsymbol{\nu}\right\} dS_1 \qquad (62)$$

with
$$\mathbf{F} = \frac{\mathbf{F}^*}{\frac{1}{2}\rho_o^* U_o^{*2} L_o^{*2}}\,.$$

where $\mathbf{v}$ denotes the velocity vector, and $\boldsymbol{\nu}$ denotes the outward directing unit vector normal to the control surface.

(A) DRAG

The drag D^* is the s-component of the total force $\mathbf{F}$. After the similar treatment to the conventional case [13], the s-component of Equation (62) can be reduced to

$$
\begin{aligned}
D = {} & \frac{1}{2\pi}\int_0^1\int_0^1 Z''(s)Z''(\xi)\log\frac{1}{|s-\xi|}\,d\xi\,ds \\
& - 2Q\int_0^1\left\{\oint_{C(s)}(\varphi+\Omega)\left(\frac{\partial\varphi}{\partial n}\right)d\tau\right\}ds \\
& - 2Q\int_0^1\oint_{C(s)}\left[\frac{\partial\varphi}{\partial n}\int_0^s\left\{\frac{\partial\varphi}{\partial n}\left(\frac{\partial\varphi}{\partial n}+\frac{\partial\Omega}{\partial n}\right)+\frac{\partial\varphi}{\partial\tau}\left(\frac{\partial\varphi}{\partial\tau}+\frac{\partial\Omega}{\partial\tau}\right)\right\}ds\right]d\tau\,ds \\
& - \frac{Q}{2\pi}\int_0^1\{Z'(s)\}^2 ds + O(Q^2, R_m^2)
\end{aligned}
$$

$$\text{for } M_0 > 1 \tag{63}$$

with $$D = \frac{D^*}{\frac{1}{2}\rho_0^* U_0^{*2} L_0^{*2}}$$

where $C(s)$ denotes the cross sectional form, and n and τ denote the outward directing normal and the counter-clockwise vectors of C, respectively. Since the drag given by Equation (63) corresponds to the integral of the hydrodynamic pressure over the surface, the Lorentz force does not make any contribution to the total drag within the accuracy of the present approximation. The first term of the right hand side of Equation (63) represents the conventional supersonic drag. In the subsonic case, this term is dropped, and other terms do not change. For example we shall calculate the drag for a slender body of revolution at a small incidence. Substituting the solutions (60a), (60b) and (60d) into Equation (63), we have

$$
\begin{aligned}
D = {} & \frac{1}{2\pi}\int_0^1\int_0^1 Z''(s)Z''(\xi)\log\frac{1}{|s-\xi|}\,d\xi\,ds \\
& - \frac{Q}{\pi}\int_0^1 Z'(s)\int_0^s Z''(\xi)\log\frac{1}{s-\xi}\,d\xi\,ds \\
& + \frac{Q}{\pi}\int_0^1\{Z'(s)\}^2\log\frac{2}{BR(s)}\,ds \\
& - 2\mu^2 Q\,\frac{R_m}{R_m+R_b}\int_0^1 Z(s)\,ds \qquad \text{for } M_0 > 1
\end{aligned}
\tag{64a}
$$

The first term of the right hand side of Equation (64a) represents the conventional supersonic drag, which is the order of $O(\tau_0)^4$. The sign of the second term corresponds to the s-distribution of the cross sectional area, and the order of this term is $O(Q\tau_0^4)$. The third term represents the mutual effect between the compressibility and the conductivity. This term always takes the positive sign and is the order of $O(Q\tau_0^4 \log \frac{1}{\tau_0})$. The last term represents the so-called induced drag, which is the order of $O(Q\tau_0^4)$. The induced drag takes the negative sign in the present case. Since the third term predominates among the last three terms of Equation (64a), the total drag increases by the electromagnetic effect. Putting $\mu = 0$ in Equation (64a), we shall obtain the drag for a body of revolution. This expression coincides with the result due to Ando[12], except only one term. The difference results from his treatment, where he omitted the last integral in the pressure formula (8a) before substituting it into Equation (62). In the conventional case, the term due to the compressible effect vanishes unless the area of the base section $Z(1)$ does not vanish. In the present case, the terms due to the electromagnetic effect still remains finite for a body with a pointed tail.

In the subsonic case, the drag is represented by

$$D = -\frac{Q}{2\pi}\int_0^1\int_0^1 Z'(s)\,Z''(\xi)\frac{(s-\xi)}{|s-\xi|}\ln\frac{1}{|s-\xi|}\,d\xi\,ds$$
$$+\frac{Q}{\pi}\int_0^1\{Z'(s)\}^2\log\frac{2}{bR(s)}\,ds - 2\mu^2 Q\frac{R_m}{R_m+R_b}\int_0^1 Z(s)\,ds \tag{64b}$$

where $b^2 = 1 - M_0^2$ $\qquad$ for $M_0 < 1$

The conventional drag vanishes. The behavior of other terms are similar to those of the supersonic case.

(B) LATERAL FORCE

By the use of Equation (62), the x-and the y-components of lateral force, X^*, Y^*, are calculated such that

$$X + iY = 2\int_0^1\oint_{c(s)}\left(\frac{\partial\varphi}{\partial s} - S\frac{\partial\psi}{\partial s}\right)e^{i\xi}\,d\tau\,ds + \int_0^1\oint_{c(s)}\left\{\left(\frac{\partial\varphi}{\partial\tau}\right)^2+\left(\frac{\partial n}{\partial\tau}\right)^2\right\}e^{i\xi}\,d\tau\,ds$$
$$+2Q\int_0^1\oint_{c(s)}e^{i\xi}\int_0^s\left\{\frac{\partial\varphi}{\partial n}\left(\frac{\partial\psi}{\partial n}+\frac{\partial\Omega}{\partial n}\right)+\frac{\partial\psi}{\partial\tau}\left(\frac{\partial\varphi}{\partial\tau}+\frac{\partial\Omega}{\partial\tau}\right)\right\}ds\,d\tau\,ds$$
$$+O(Q^2, R_m^2) \tag{65a}$$

with $\qquad X = \dfrac{X^*}{\frac{1}{2}\rho_0^* U_0^{*2} L_0^{*2}} \qquad\qquad Y = \dfrac{Y^*}{\frac{1}{2}\rho_0^* U_0^{*2} L_0^{*2}}$

where ξ denotes the angle between the normal vector n of the curve C and the x-axis.

Applying Equation (65a) to the body of revolution at a small incidence, μ, we obtain

$$X = -2a\mu\left\{2 - \frac{R_m}{R_m + R_b}\right\}\int_0^1 Z(s)\,ds$$
$$Y = 0 \tag{65b}$$

The equation (65b) implies that the lift takes the negative sign for a positive value of the incidence, μ. This negative lift corresponds to the negative induced drag discussed above. For the sake of considering the physical meaning of the negative lift, we shall decompose the lift to the two integrals of the pressure and the Maxwell stress tensor over the surface of the body. The integral of the pressure is given by

$$-\int_0^1\int_0^{2\pi} C_p \cos\theta\, R(s)\,d\theta\,ds = -2a\mu\int_0^1 Z(s)\,ds \tag{66a}$$

The integral of Maxwell stress tensor is given by

$$-2S\int_0^1\int_0^{2\pi} h_1 \cos\theta\, R(s)\,d\theta\,ds = -\frac{2a_b\mu}{1+\frac{R_b}{R_m}}\int_0^1 Z(s)\,ds \tag{66b}$$

First we shall consider an insulated body. We shall suppose $Z'(s)$ is positive for $0 < s < s_1$, and negative for $s_1 < s < 1$. For the region, where $Z'(s)$ is positive, the velocity field in the sectional plane consists of the dipole like and the diverging radial fields (c.f. Fig. 6). As a result, the induced current flows clockwise around the circular section C. The Lorentz force due to the correlation between the induced current and the uniform applied magnetic field acts in the inward direction normal to the circle. Remembering the velocity field shown in Fig. 6, it is realized that the velocity due to the dipole and the Lorentz force take the similar directions with each other on the upside of the body, but they take the opposite directions on the downside. We shall consider a couple of small volume elements of fluid flowing along stream lines of the upside and the downside of the body. The work performed by the Lorentz force when the fluid passes along the stream line of the upside from 0 to s $(s < s_1)$, should be larger than the work performed when the fluid passes along the stream line of the downside. Thus the pressure on the upside is larger than that on the downside for the same section, s. After the position s_1, $Z'(s)$ is negative. The velocity field in the sectional plane consists of the dipole like field and the converging radial field. Then the induced current flows counterclockwise, and the Lorentz force acts in the outward direction. As a result, the work performed on the upside is smaller than the work performed on the downside. Thus the excess pressure on the upside decreases as s increases $(s_1 < s)$, and the pressures of the both sides take the same value at the tail, $s = 1$. Since the upside pressure is larger than the downside pressure, the integral of the pressure distribution takes the negative sign (c.f. Equation (66a)).

Next we shall consider a conducting body. Remembering Equation (60e), we have the inside current density such as

$$j_{xi} = 0 \qquad\qquad j_{yi} = -\frac{R_b}{1+\frac{R_b}{R_m}}\mu \tag{67}$$

The Lorentz force due to the correlation between the uniform horizontal induced current and the applied magnetic field acts downward in the vertical direction (c.f. Equation (66b)). Thus, both the integrals of the pressure and the Maxwell stress tensor contribute to the negative lift.

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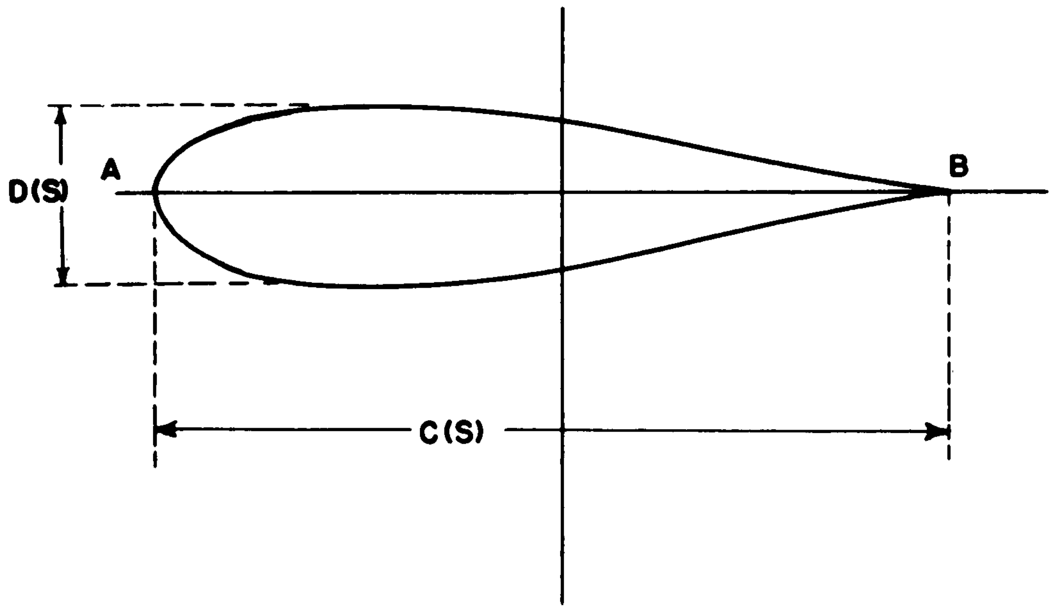

Figure 1 Cross section of slender body in z-plane: Symmetric Joukowski aerofoil

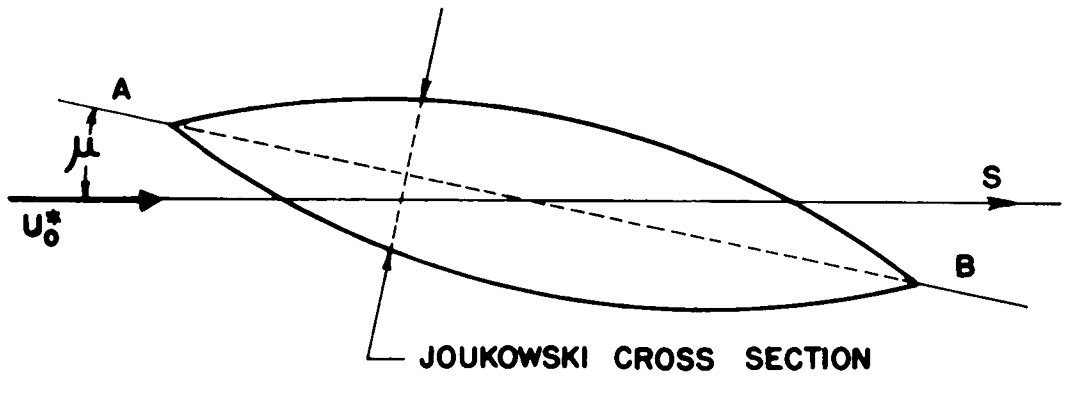

Figure 2 Meridian section of slender body at small incidence

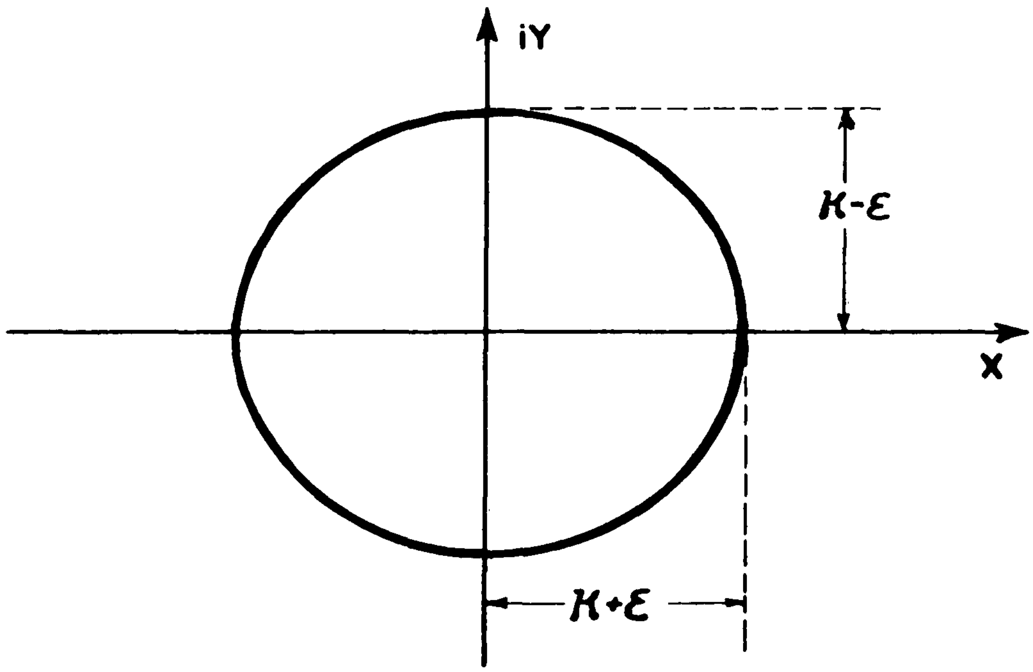

Figure 3 Cross section of slender body in z-plane: Ellipse

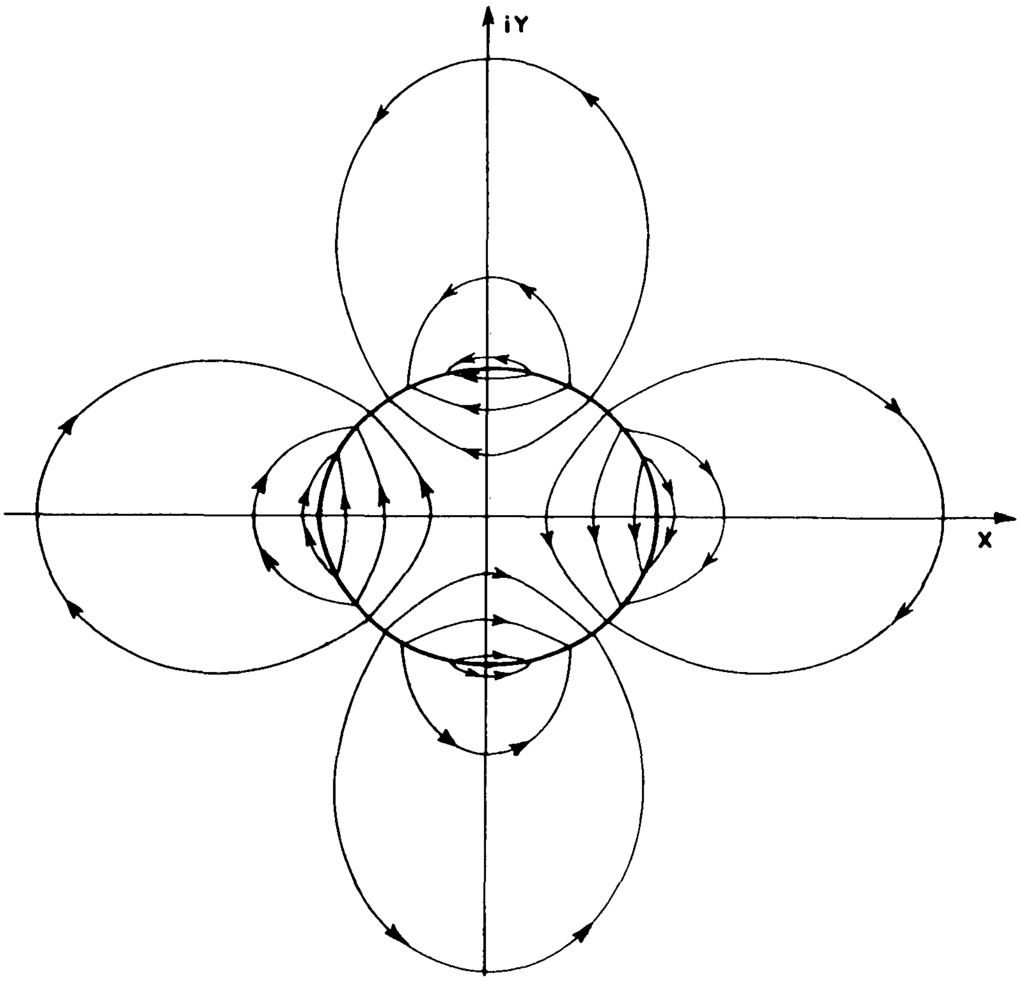

Figure 4 Induced electric field for body of elliptic cross section

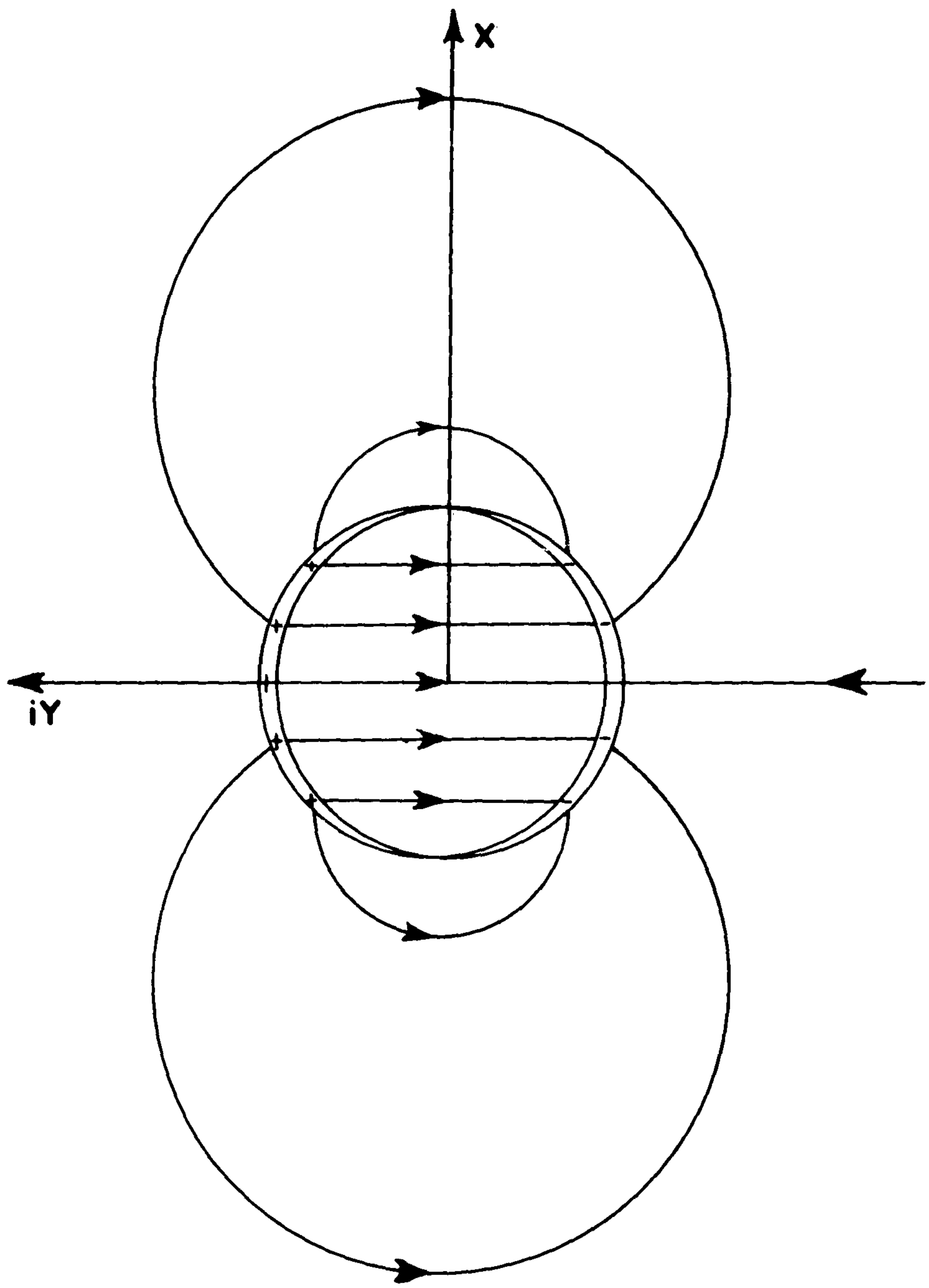

Figure 5 Induced electric field and surface charge distribution for body of revolution at small incidence

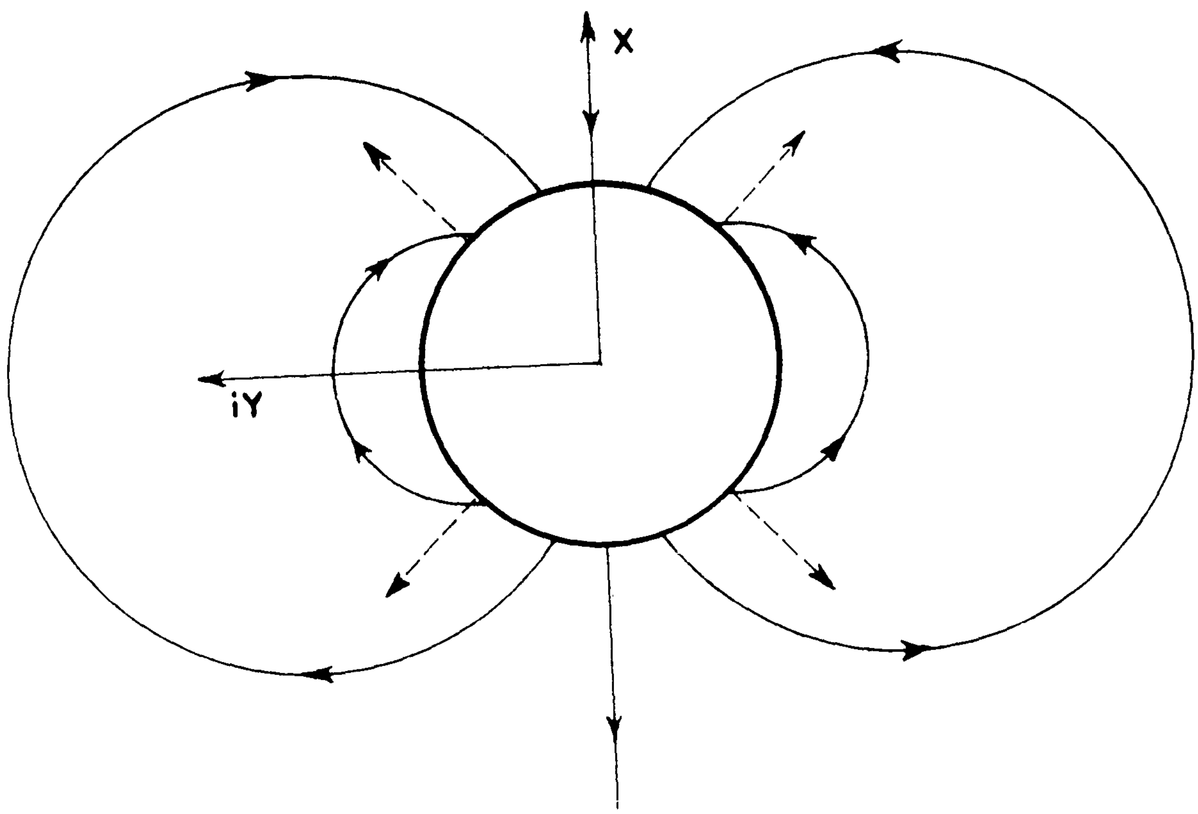

Figure 6 Perturbation velocity field in z-plane for body of revolution at small incidence